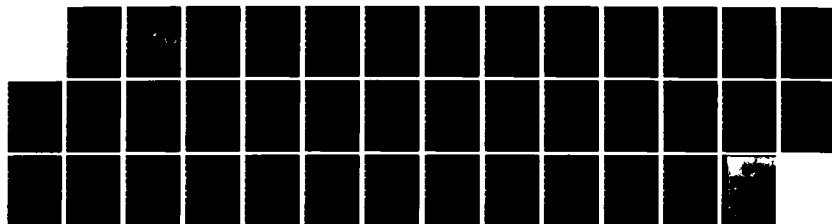SUPPRESSION OF FINITE-AMPLITUDE EFFECTS IN SLOSHING
MODES IN CYLINDRICAL CAVITIES(U) NAVAL POSTGRADUATE
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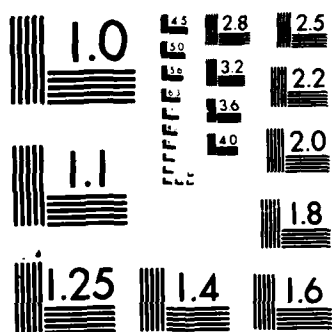MICROCOPY RESOLUTION TEST CHART

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THESIS

SUPPRESSION OF FINITE-AMPLITUDE EFFECTS
IN SLOSHING MODES IN CYLINDRICAL CAVITIES

by

Si Hwan Yum

December 1983

Thesis Advisor: Alan B. Coppens

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Suppression of Finite-Amplitude Effects in Sloshing
Modes in Cylindrical Cavities

by

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN ENGINEERING ACOUSTICS

from the

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ABSTRACT

A pertubation expansion is formulated for the three-dimensional, nonlinear, acoustic-wave equation with dissipative term describing the viscous and thermal energy losses encountered in a cylindrical cavity. The theoretical results show that nonlinear effects in sloshing modes are strongly suppressed.

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LIST OF SYMBOLS

c phase velocity in the cavity

$c^2 \square^2$ $= c^2 \nabla^2 - \partial^2/\partial t^2$

c_0 $(dP/d\rho)^{\frac{1}{2}}$ at $\rho = \rho_0$

c_n effective phase speed associated with a standing wave at resonance

c_p specific heat at constant pressure

k W/c, propagation constant associated with a standing wave

M peak Mach number of the driven standing wave

p $= P - P_0 = -\rho_0 \, \partial\Phi/\partial t = $ acoustic pressure

P, P_0 instantaneous and equilibrium total pressure in the field

T_K absolute temperature in kelvin

U particle velocity

U_n particle speed of the ηth-order perturbation solution

α infinitesimal-amplitude attenuation constant

$\alpha_n c_0$ temporal decay constant of a resonance

β $= (\gamma + 1)/2$ for a gas

γ $= c_p/c_v$, ratio of specific heats for a gas

ρ, ρ_0 instantaneous and equilibrium densities

Φ velocity potential

W $= 2\pi f$, (angular) grequency at which the cavity is driven

W_n (angular) frequency of a resonance

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I. <u>INTRODUCTION</u>

The topic of finite amplitude acoustic standing waves in sloshing modes of a cylindrical cavity is interesting theoretically, but development of the subject has not been extensive. The purpose of this research is to present the results of a power series perturbation approach to the problem.

II. THE NON-LINEAR WAVE EQUATION

A. GENERAL

It is well known [1] that for α/k and $M \ll 1$, where
measures the fractional loss per wavelength and M is the peak
Mach number of the source, loss terms and nonlinear terms in
the constitutive equations can be separately approximated with
the help of linear, lossless acoustic relations. The force
equation appropriate for acoustical processes in systems for
which gravitational effects are unimportant is

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} + \frac{1}{\rho}\nabla P = \frac{1}{\rho}\mathcal{L}\vec{u} \qquad (2\text{-}1)$$

where $\vec{u} = u\hat{u}$ is the particle velocity, ρ is the instantaneous
density of the fluid, P is the instantaneous total pressure in
the field, and the operator $\mathcal{L}$ symbolically describes those
physical processes leading to absorption and dispersion. We
used two additional equations. The first is the equation of
state for a perfect gas

$$P = \rho r T_K \qquad (2\text{-}1\text{-}a)$$

where r is a constant whose value depends on the particular
gas involved and T_K is the absolute temperature in Kelvin.
The second is the continuity equation

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$$\frac{\partial s}{\partial t} + \nabla \cdot \left[(1+s)\vec{U} \right] = 0 \qquad (2\text{-}1\text{-}b)$$

where $s = (\rho - \rho_o)/\rho_o$ is the condensation at any point and ρ is the equilibrium density of the fluid. If we ignore rotational effects, then

$$\vec{U} = \nabla \Phi \qquad (2\text{-}2)$$

where Φ is the velocity potential. Combination of Eqs.(2-1)-(2-2) and the neglect of terms of orders higher then M^2, $M(\alpha/k)$, and $(\alpha/k)^2$, yields a quadratically nonlinear wave equation,

$$\left(c_o^2 \square^2 + \frac{\partial}{\partial t} \mathcal{L} \right) \frac{p}{\rho_o c_o^2} = -\frac{1}{2} \frac{\partial^2}{\partial t^2} \left[\gamma \left(\frac{p}{\rho_o c_o^2} \right)^2 + \left(\frac{U}{c_o} \right)^2 \right]$$
$$+ \frac{1}{2} c_o^2 \nabla^2 \left[\left(\frac{p}{\rho_o c_o^2} \right)^2 - \left(\frac{U}{c_o} \right)^2 \right] \qquad (2\text{-}3)$$

where $c_o^2 = (\partial p/\partial \rho)$ (adiabatic), p=acoustic pressure, $U^2 = \vec{U} \cdot \vec{U}$, γ is the ratio of heat capacities, and

$$c_o^2 \square^2 = c_o^2 \nabla^2 - \frac{\partial^2}{\partial t^2} \qquad (2\text{-}4)$$

The left-hand side of Eq.(2-3) is the classical, linear wave equation with losses pertinent to the system under study.

The right-hand side can be interpreted as a forcing function consisting of a three-dimensional spatial distribution of phase-coherent sources.

In a second-order perturbation theory, this volume forcing function is obtained from the classical(first-order) solution P_1 of the acoustical problem. The second-order perturbation solution P_1+P_2 describes the non-linearities P_2 resulting from the self interaction of the classical solution P_1. Higher-order perturbation solutions consider the interaction of the nonlinear solution with itself, and the forcing function is composed of products of both classical and non-linearly generated terms. Thus, if a system is driven at frequency w, the non-linear term in Eq.(2-3) will force the existence of all integer multiples nw of the driving frequeney and the full solution must contain all harmonics of the input frequency. In a closed cavity, each of those nonlinearly generated waves whose frequency lies near the resonance frequency of a standing wave of the cavity and whose assciated spatial function matches that of the standing wave can be strongly excited [2].

As far as the author has been able to determine, there has been only one previous study of this system published in the open literature. This was by Maslen and Moore in 1956 [3]. Their approach resulted in a series expansion which did not converge if the relevant nomal mode frequencies were integerally related. Their interpretation of the quenching of the nonlinear effect was based on the "scattering effect of the wall".

We feel our interpretation based upon nonlinearally generated volume sources stimulating the allowed standing waves of the cavity is more accessible and informative. Further, the mathematical approach developed herein appear to avoid any difficulties in convergence. Their conclusion that high amplitude monofrequency transverse oscillations can exist is consistent with our finding.

B. APPLICATION TO THE CYLINDRICAL CAVITY

The circular cylinder is provided with a "point" source of sound. By properly positioning the cylinder with respect to the sound source it is possible to effectively drive the enclosed air into various modes of vibration. The rigid-walled cylinder forces the component of particle velocity perpendicular to each cavity surface to vanish at the surface. The resulting steady-state solution to the linear wave equation in cylindrical coordinates is

$$P(r, \varphi, \mathfrak{z}) = A_{nm\ell} \cos(k_{\mathfrak{z}\ell}\, \mathfrak{z}) \cos(n\varphi) \cos(W_{nm\ell}\, t) J_n(k_{rnm}\, r)$$

(2-5)

where J_n are the cylindrical Bessel functions and application of the boundary condition to the sides yields

$$k_{rnm} = \frac{J_{nm}}{a}$$

(2-5-a)

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where a is the radius of cylinder and j_{nm} are the arguments of the exterema of the $\eta^{\underline{th}}$ Bessel function.

The nomal mode frequency is dependent on $k_{3\ell}$ and k_{rmm} ,

$$k_{rmm}^z + k_{3\ell}^2 \doteq \frac{w_{nmk}^2}{c_o^2} \qquad (2\text{-}5\text{-}b)$$

The standing wave well be identified by the ordered integers (n, m, ℓ) describing its spatial dependence.

1. Symmetric Modes

The simplest waves in cylindrical coordinates are those that depend only on the distance r from Z-axis, the gradient takes the form

$$\nabla = \hat{y}\, k\, \frac{\partial}{\partial y} \qquad (2\text{-}6)$$

where

$$y = k\, r \qquad (2\text{-}6\text{-}a)$$

and k=w/c is the wave number or propagation constant. The Laplacian becomes

$$\nabla^z = k^2 \left(\frac{\partial^2}{\partial y^2} + \frac{1}{y}\frac{\partial}{\partial y} \right) \qquad (2\text{-}6\text{-}b)$$

The suitable trial solution appropriate for symmetric modes is the linear solution(0, m, 0) of frequency w

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$$\frac{P_1}{\rho_0 C_0^2} = M J_0(y) \sin \omega t \qquad ($$

The velocity potential for this limit can be defined as

$$\Phi = \left(\frac{C_0^2}{\omega}\right) M J_0(y) \cos \omega t \qquad ($$

and $\vec{U_1}/C_0$ is

$$\frac{\vec{U_1}}{C_0} = \hat{y} M J_0'(y) \cos \omega t = -\hat{y} M J_1 \cos \omega t \qquad ($$

Because we assume that the surfaces of the cavity are rigid then at r=a the appropriate boundary condition is $J_0'=0$. Thus, we have $ka = j'_{0m}$.

To generate the second order solution, we first note that

$$\left(\frac{P_1}{\rho_0 C_0^2}\right)^2 = \frac{1}{2} M^2 J_0^2 (1 - \cos 2\omega t) \qquad (2$$

and

$$\left(\frac{U_1}{C_0}\right)^2 = \frac{1}{2} M^2 J_1^2 (1 + \cos 2\omega t) \qquad (2$$

Next, the second derivatives of Eqs.(2-10) and (2-11) with respect to time are

$$\frac{\partial^2}{\partial t^2}\left(\frac{P_1}{\rho_0 C_0^2}\right)^2 = \frac{(zw)^2}{z} M^2 J_c^2 \cos zwt \qquad (2\text{-}12)$$

and

$$\frac{\partial^2}{\partial t^2}\left(\frac{U_1}{C_0}\right)^2 = -\frac{(zw)^2}{z} M^2 J_1^2 \cos zwt \qquad (2\text{-}13)$$

Substituting Eqs.(2-2) and (2-13) into the first term of the
right-hand side of Eq.(2-3) yields

$$\frac{(zw)^2}{4} M^2 \cos zwt \left[J_1^2 - \gamma J_0^2 \right] \qquad (2\text{-}14)$$

and the second term of right-hand side of Eq.(2-3) becomes

$$\frac{w^2 M^2}{4}\left[(-J_z^2 + 4J_1^2 - 3J_0^2) - (J_z^2 - J_0^2)\cos zwt\right] \qquad (2\text{-}15)$$

since

$$C_0^2 \nabla^2 J_0^2 = zw^2 (J_1^2 - J_0^2) \qquad (2\text{-}16)$$

or

$$C_0^2 \nabla^2 J_1^2 = zw^2 (\frac{1}{z} J_z^2 + \frac{1}{z} J_0^2 - J_1^2) \qquad (2\text{-}17)$$

With the use of Eqs.(2-14) and (2-15) we can write the right-

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hand side of Eq.(2-3) as

$$\frac{(2w)^2}{4} M^2 \left\{ \left[-\frac{1}{4} J_2^2 + J_1^2 - \frac{3}{4} J_0^2 \right] + \left[-\frac{1}{4} J_2^2 + J_1^2 - (\gamma - \frac{1}{4}) J_0^2 \right] \right.$$

$$\left. \cdot \cos 2wt \right\} \qquad (2-18)$$

and the appropriate inhomogeneous wave equation for the second
order perturbation solution is

$$\left(C_0^2 \Box^2 + \frac{\partial}{\partial t} \mathcal{L} \right) \frac{P_2}{\rho_0 C_0^2} = \frac{(2w)^2}{4} M^2 \left\{ \left[-\frac{1}{4} J_2^2 + J_1^2 - \frac{3}{4} J_0^2 \right] + \left[-\frac{1}{4} J_2^2 \right. \right.$$

$$\left. \left. + J_1^2 - (\gamma - \frac{1}{4}) J_0^2 \right] \cos 2wt \right\} \qquad (2-19)$$

2. Non-Symmetric Modes

The non planar waves in cylindrical coordinates are
those that depend on the distance r from Z axis and the angle
φ from X-axis. The gradient takes the form

$$\nabla = k \cdot \left(\hat{y} \frac{\partial}{\partial y} + \hat{\varphi} \frac{1}{y} \frac{\partial}{\partial \varphi} \right) \qquad (2-20)$$

where

$$y = k r$$

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and the Laplacian becomes

$$\nabla^2 = k^2 \left(\frac{\partial^2}{\partial y^2} + \frac{1}{y} \frac{\partial}{\partial y} + \frac{1}{y^2} \frac{\partial^2}{\partial \varphi^2} \right) \qquad (2\text{-}20\text{-}a)$$

The suitable trial solution appropriate to the forcing function of frequency w for exciting the (n, m, 0) standing wave is

$$\frac{P_1}{\rho_0 c_0^2} = M J_n(k_{nm} r) \cos n\varphi \sin wt, \quad k_{nm} = j'_{nm} / a \qquad (2\text{-}21)$$

where $J_n(kr)$ is the Bessel function of the first kind and order n. The velocity potential is approximated by

$$\Phi_1 = \left(\frac{c_0^2}{w} \right) M J_n(kr) \cos n\varphi \cos wt \qquad (2\text{-}22)$$

and $\vec{U}_1 / c_0$ is

$$\frac{\vec{U}_1}{c_0} = M \left(\hat{y} J_n' \cos n\varphi \cos wt - \hat{\varphi} \frac{n}{y} J_n \sin n\varphi \cos wt \right) \qquad (2\text{-}23)$$

Thus,

$$\left(\frac{P_1}{\rho_0 c_0^2} \right)^2 = M^2 J_n^2 \cos^2 n\varphi \sin^2 wt \qquad (2\text{-}24)$$

or

$$\left(\frac{P_1}{\rho_0 c_0^2} \right)^2 = \frac{1}{4} M^2 (\quad \text{-} \quad wt)(J_n^2 + J_n^2 \cos 2n\varphi) \qquad (2\text{-}25)$$

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and

$$\left(\frac{U_1}{C_0}\right)^2 = \frac{1}{4} M^2 (1 + \cos 2\omega t) \left[\frac{1}{2}\left(J_{n+1}^2 + J_{n-1}^2\right) - J_{n+1}J_{n-1}\cos 2n\varphi\right]$$

(2-26)

and the second derivative of Eqs.(2-25) and (2-26) with respect to time is

$$\frac{\partial^2}{\partial t^2}\left(\frac{P_1}{\rho_0 C_0^2}\right)^2 = \frac{(2\omega)^2}{4} M^2 J_n^2 (1 + \cos 2n\varphi)\cos 2\omega t$$

(2-27)

and

$$\frac{\partial^2}{\partial t^2}\left(\frac{U_1}{C_0}\right)^2 = -\frac{(2\omega)^2}{4} M^2 \left\{\left[J_n'^2 + \left(\frac{n}{y}\right)^2 J_n^2\right]\right.$$

$$\left. + \left[J_n'^2 - \left(\frac{n}{y}\right)^2 J_n^2\right]\cos 2n\varphi\right\}$$

(2-28)

Substituting Eqs.(2-26) and (2-27) into the first term of Eq.(2-3) yields

$$\frac{(2\omega)^2}{8} M^2 \cos 2\omega t \left\{\left[J_n'^2 + \left(\frac{n}{y}\right)^2 J_n^2 - \gamma J_n^2\right]\right.$$

$$\left. + \left[J_n'^2 - \left(\frac{n}{y}\right)^2 J_n^2 - \gamma J_n^2\right]\cos 2n\varphi\right\}$$

(2-29)

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or

$$\frac{(zW)^3}{8} M^2 \cos zWt \left\{ \left[\frac{1}{z} (J_{n+1}^z + J_{n-1}^z) - \gamma J_n^z \right] \right.$$

$$\left. - \left[J_{n+1} J_{n-1} + \gamma J_n^z \right] \cos zn\varphi \right\} \qquad (2\text{-}30)$$

since

$$J_n'^z = \frac{1}{4} \left(J_{n+1}^z + J_{n-1}^z - z J_{n+1} J_{n-1} \right) \qquad (2\text{-}31)$$

and

$$\frac{n^2}{y^2} J_n^2 = \frac{1}{4} \left(J_{n+1}^2 + J_{n-1}^2 + z J_{n+1} J_{n-1} \right) \qquad (2\text{-}32)$$

In order to solve the second term of Eq.(2-3), we can use

$$C_0^2 \nabla^2 J_n^z = zW^2 \left(\frac{1}{z} J_{n+1}^z + \frac{1}{z} J_{n-1}^z - J_n^2 \right) \qquad (2\text{-}33)$$

$$C_0^z \nabla^z J_n^z \cos 2n\varphi = zW^2 (-J_{n+1} J_{n-1} - J_n^2) \cos 2n\varphi \qquad (2\text{-}34)$$

$$C_0^2 \nabla^z J_{n+1} J_{n-1} \cos zn\varphi = zW^2 (J_{n+1}' J_{n-1}' - J_n^2$$

$$- J_{n+2} J_{n-z} + J_{n+z} J_n) \cos zn\varphi \qquad (2\text{-}35)$$

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since

$$J_n'^2 + \left(\frac{n}{y}\right)^2 J_n^2 = \frac{1}{2}\left(J_{n+1}^2 + J_{n-1}^2\right) \tag{2-36}$$

and

$$J_n' - \left(\frac{n}{y}\right)^2 J_n^2 = -J_{n+1} J_{n-1} \tag{2-37}$$

Thus, Substituting Eqs.(2-33) through(2-35) into the second term of Eq.(2-3) yields

$$\frac{1}{2} C_0^2 \nabla^2 \left(\frac{\rho_1}{\rho_0 C_0^2}\right)^2 = \frac{1}{4} M^2 w^2 (1 - \cos 2wt)\left[\left(\frac{1}{2} J_{n+1}^2\right.\right.$$

$$\left.\left. + \frac{1}{2} J_{n-1}^2 - J_n^2\right) + \left(-J_{n+1} J_{n-1} - J_n^2\right) \cos 2n\varphi\right] \tag{2-38}$$

and

$$\frac{1}{2} C_0^2 \nabla^2 \left(\frac{U_1}{C_0}\right)^2 = \frac{1}{4} M^2 w^2 (1 + \cos 2wt)\left[\frac{1}{2}\left(\frac{1}{2} J_{n+2}^2\right.\right.$$

$$\left. - J_n^2 - J_{n+1}^2 + \frac{1}{2} J_{n-2}^2 - J_{n-1}^2\right) + \left(\frac{1}{2} J_n^2\right.$$

$$\left.\left. + \frac{1}{2} J_{n+2} J_{n-2} + J_{n+1} J_{n-1}\right) \cos 2n\varphi\right] \tag{2-39}$$

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With Eqs.(2-30), (2-38) and (2-39) we can get the inhomogeneous wave equation for the second order perturbation as

$$\left(c_0^2 \square^2 + \frac{\partial}{\partial t} \mathcal{L} \right) \frac{P_2}{\rho_0 c_0^2} = \frac{(2W)^2}{4} M^2 \left[\frac{1}{4} \left(J_{n+1}^z + J_{n-1}^z - \frac{3}{2} J_n^z \right. \right.$$

$$\left. - \frac{1}{4} J_{n+2}^z - \frac{1}{4} J_{n-2}^z \right) + \frac{1}{4} \left(-2 J_{n+1} J_{n-1} \right.$$

$$\left. - \frac{3}{2} J_n^2 - \frac{1}{2} J_{n+2} J_{n-2} \right) \cos 2n\varphi \Big]$$

$$+ \frac{(2W)^2}{4} M^2 \cos 2Wt \left\{ \left[-\frac{1}{16} J_{n+2}^z + \frac{1}{4} J_{n+1}^z \right. \right.$$

$$\left. - \left(\frac{\gamma}{2} - \frac{1}{8} \right) J_n^z + \frac{1}{4} J_{n-1}^z - \frac{1}{16} J_{n-2}^2 \right]$$

$$- \left[\frac{1}{8} J_{n+2} J_{n-2} + \frac{1}{2} J_{n+1} J_{n-1} + \left(\frac{\gamma}{2} \right. \right.$$

$$\left. \left. \left. - \frac{1}{8} \right) J_n^2 \right] \cos 2n\varphi \right\} \qquad (2\text{-}40)$$

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If we let n=0 in Eq(2-40), it reduces to Eq.(2-19), as it must. Substituting n=1 into Eq.(2-40) yields the equation with forcing term resulting from the (1, m, 0) sloshing mode,

$$
\left(C_0^2 \Box^2 + \frac{\partial}{\partial t} \mathcal{L} \right) \frac{P_2}{\rho_0 C_0^2} = \frac{(2w)^2}{4} M^2 \left[\frac{1}{4} \left(-\frac{1}{4} J_3^2 + J_2^2 \right. \right.
$$

$$
\left. -\frac{7}{4} J_1^2 + J_0^2 \right) + \frac{1}{4} \left(-\frac{3}{2} J_1^2 + \frac{1}{2} J_3 J_1 \right.
$$

$$
\left. \left. -2 J_2 J_0 \right) \cos 2\varphi \right] + \frac{(2w)^2}{4} M^2 \cos 2wt \left\{ \vphantom{\frac{1}{4}} \right.
$$

$$
\left[-\frac{1}{16} J_3^2 + \frac{1}{4} J_2^2 - \left(\frac{\gamma}{2} - \frac{1}{16} \right) J_1^2 + \frac{1}{4} J_0^2 \right]
$$

$$
+ \left[\frac{1}{8} J_3 J_1 - \frac{1}{2} J_2 J_0 - \left(\frac{\gamma}{2} - \frac{1}{8} \right) J_1^2 \right] \cos 2\varphi \right\}
$$

(2-41)

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III. METHOD OF SOLUTION

Recall that the equation with forcing term resulting from the $(1, m, 0)$ sloshing mode can be written as a function of frequency,

$$\frac{(2W)^2}{4} M^2 \cos 2\omega t \left\{ \left[-\frac{1}{16} J_3^2 + \frac{1}{4} J_2^z - \left(\frac{\gamma}{2} - \frac{1}{16} \right) J_1^z + \frac{1}{4} J_0^z \right] \right.$$
$$\left. + \left[\frac{1}{8} J_3 J_1 - \frac{1}{2} J_2 J_0 - \left(\frac{\gamma}{2} - \frac{1}{8} \right) J_1^z \right] \cos 2\varphi \right\} \quad (3\text{-}1)$$

and the left-hand side of Eq.(2-3) is

$$\left[C_0^z k^z \left(\frac{\partial^2}{\partial y^2} + \frac{1}{y} \frac{\partial}{\partial y} \right) - \frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial t} \mathcal{L} \right] \frac{P_z}{\rho_0 C_0^z} \quad (3\text{-}1\text{-}a)$$

If the harmonics of the frequency at which the cavity is driven are not close to any resonant frequencies, we can ignore the lossy term. So, Eq.(3-1-à) can be written as

$$\left[C_0^z k^z \left(\frac{\partial^2}{\partial y^2} + \frac{1}{y} \frac{\partial}{\partial y} \right) - \frac{\partial^2}{\partial t^2} \right] \frac{P_z}{\rho_0 C_0^z} \quad (3\text{-}2)$$

Let us assume that the solution of Eq.(3-1) can be expressed as

$$\frac{P}{\rho_0 C_0^z} = M^2 \cos 2\omega t \left[U_n(y) + V_n(y) \cos 2n\varphi \right] \quad (3\text{-}3)$$

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and define

$$\frac{P_{nv}}{\rho_0 c_i^2} = M^2 V_n(kr) \cos zn\phi \cos z\omega t \qquad (3-4)$$

and

$$\frac{P_{nu}}{\rho_0 c_i^2} = M^2 U_n(kr) \cos z\omega t \qquad (3-5)$$

Combination of Eqs.(3-2) and (3-3) yields

$$\left[\left(\frac{c_2 k}{z\omega}\right)^2\left(\frac{\partial^2}{\partial y^2} + \frac{1}{y}\frac{\partial}{\partial y}\right) + 1\right] U_n(y) = \frac{1}{4}\left[-\frac{1}{16}J_{n+z}^z + \frac{1}{4}J_{n+1}^z\right.$$

$$\left. -\left(\frac{\gamma}{z} - \frac{1}{8}\right)J_n^z + \frac{1}{4}J_{n-1}^z - \frac{1}{16}J_{n-z}^z\right] \qquad (3-6)$$

and

$$\left\{\left(\frac{c_2 k}{z\omega}\right)^2\left[\frac{\partial^2}{\partial y^2} + \frac{1}{y}\frac{\partial}{\partial y} - \left(\frac{zn}{y}\right)^2\right] + 1\right\} V_n(y) = \frac{1}{4}\left[-\frac{1}{8}J_{n+z}J_{n-z}\right.$$

$$\left. -\frac{1}{z}J_{n+1}J_{n-1} - \left(\frac{\gamma}{z} - \frac{1}{8}\right)J_n^z\right] \qquad (3-7)$$

A. POWER SERIES METHOD

In order to solve the right-hand side of Eq.(3-7), we can use the definition [4]

$$J_\nu(y) J_u(y) = \left(\frac{1}{2}y\right)^{\nu+u} \sum_{k=0}^{\infty} \cdot$$

$$\frac{(-1)^k \Gamma(\nu+u+2k+1)\left(\frac{1}{2}y\right)^{2k}}{\Gamma(\nu+k+1)\Gamma(u+k+1)\Gamma(\nu+u+k+1)\, k!} \tag{3-8}$$

where $\Gamma(z+1) = z!$

1. Coefficient

a. Substituting $\nu = u = n+2$ into Eq.(3-8) yields

$$J_{n+2}^2(y) = \left(\frac{1}{2}y\right)^{2n+4} \sum_{k=0}^{\infty}$$

$$\frac{(-1)^k (2n+2k+4)!\left(\frac{1}{2}y\right)^{2k}}{\left[(n+k+2)!\right]^2 (2n+k+4)!\, k!} \tag{3-9}$$

or

$$J_{n+2}^2(y) = \sum_{k=0}^{\infty} B_{nk}\left(\frac{1}{2}y\right)^{2(n+k+2)} \tag{3-10}$$

where

$$B_{nk} = (-1)^k \frac{(2n+2k+4)!}{\left[(n+k+2)!\right]^2 (2n+k+4)!\, k!} \tag{3-11}$$

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b. Substituting $\nu = n+2$, $\mu = n-2$ into Eq.(3-8) yields

$$J_{n+2}(y)\, J_{n-2}(y) = \left(\tfrac{1}{2}y\right)^{2n} \sum_{k=0}^{\infty} \cdot$$

$$\frac{(-1)^k\,(2n+2k)!\,\left(\tfrac{1}{2}y\right)^{2k}}{(n+k+2)!\,(n+k-2)!\,(2n+k)!\,k!} \qquad (3\text{-}12)$$

or

$$J_{n+2}(y)\, J_{n-2}(y) = \sum_{k=0}^{\infty} C_{nk}\left(\tfrac{1}{2}y\right)^{2(n+k)} \qquad (3\text{-}13)$$

where

$$C_{nk} = (-1)^k \frac{(2n+2k)!}{(n+k+2)!\,(n+k-2)!\,(2n+k)!\,k!} \qquad (3\text{-}14)$$

c. Substituting $\nu = n+1$, $\mu = n-1$ into Eq.(3-8) yields

$$J_{n+1}(y)\, J_{n-1}(y) = \left(\tfrac{1}{2}y\right)^{2n} \sum_{k=0}^{\infty} \cdot$$

$$\frac{(-1)^k\,(2n+2k)!\,\left(\tfrac{1}{2}y\right)^{2k}}{(n+k+1)!\,(n+k-1)!\,(2n+k)!\,k!} \qquad (3\text{-}15)$$

or

$$J_{n+1}(y)\, J_{n-1}(y) = \sum_{k=0}^{\infty} D_{nk}\left(\tfrac{1}{2}y\right)^{2(n+k)} \qquad (3\text{-}16)$$

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where

$$D_{nk} = (-1)^k \frac{(2n+2k)!}{(n+k+1)!\,(n+k-1)!\,(2n+k)!\,k!} \qquad (3-17)$$

d. Substituting $\nu = u = n$ into Eq.(3-8) yields

$$J_n^2(y) = \left(\tfrac{1}{2}y\right)^{2n} \sum_{k=0}^{\infty} \frac{(-1)^k (2n+2k)!\,\left(\tfrac{1}{2}y\right)^{2k}}{\left[(n+k)!\right]^2 (2n+k)!\,k!} \qquad (3-18)$$

or

$$J_n^2(y) = \sum_{k=0}^{\infty} E_{nk} \left(\tfrac{1}{2}y\right)^{2(n+k)} \qquad (3-19)$$

where

$$E_{nk} = (-1)^k \frac{(2n+2k)!}{\left[(n+k)!\right]^2 (2n+k)!\,k!} \qquad (3-20)$$

e. Substituting $\nu = u = n+1$ into Eq.(3-8) yields

$$J_{n+1}^2(y) = \left(\tfrac{1}{2}y\right)^{2n+2} \sum_{k} \cdot$$

$$\frac{(-1)^k (2n+2k+2)!\,\left(\tfrac{1}{2}y\right)^{2k}}{\left[(n+k+1)!\right]^2 (2n+k+2)!\,k!} \qquad (3-21)$$

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or

$$J_{n+1}^2(y) = \sum_{k=0}^{\infty} G_{nk} \left(\tfrac{1}{2}y\right)^{2(n+k+1)}$$ (

where

$$G_{nk} = (-1)^k \frac{(2n+2k+2)!}{\left[(n+k+1)!\right]^2 (2n+k+2)! \, k!}$$ (

f.   Substituting $\nu = \mu = n-1$ into Eq.(3-8) yields

$$J_{n-1}^2(y) = \left(\tfrac{1}{2}y\right)^{2n-2} \sum_{k=0}^{\infty} \cdot$$

$$\frac{(-1)^k (2n+2k-2)! \left(\tfrac{1}{2}y\right)^{2k}}{\left[(n+k-1)!\right]^2 (2n+k-2)! \, k!}$$ (

or

$$J_{n-1}^2(y) = \sum_{k=0}^{\infty} H_{nk} \left(\tfrac{1}{2}y\right)^{2(n+k-1)}$$ (

where

$$H_{nk} = (-1)^k \frac{(2n+2k-2)!}{\left[(n+k-1)!\right]^2 (2n+k-2)! \, k!}$$ (

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g. Substituting $\nu = \mu = n-2$ into Eq.(3-8) yields

$$J_{n-2}^2(y) = \left(\tfrac{1}{2}y\right)^{2n-4} \sum_{k=0}^{\infty} \cdot$$

$$\frac{(-1)^k (2n+2k-4)! \left(\tfrac{1}{2}y\right)^{2k}}{\left[(n+k-2)!\right]^2 (2n+k-4)! \, k!} \tag{3-27}$$

or

$$J_{n-2}^2(y) = \sum_{k=0}^{\infty} \Theta_{nk}\left(\tfrac{1}{2}y\right)^{2(n+k-2)} \tag{3-28}$$

where

$$\Theta_{nk} = (-1)^k \frac{(2n+2k-4)!}{\left[(n+k-2)!\right]^2 (2n+k-4)! \, k!} \tag{3-29}$$

2. $\underline{V_n(y)}$

With the help of Eqs.(3-9) through(3-29) Eq.(3-7) can
be written as

$$\left\{ \left(\tfrac{1}{2a}\right)^2 \left[\frac{\partial^2}{\partial y^2} + \frac{1}{y}\frac{\partial}{\partial y} - \left(\frac{2n}{y}\right)^2 \right] + 1 \right\} V_n(y)$$

$$= \frac{1}{4} \sum_{k=0}^{\infty} \left[-\frac{1}{8}C_{nk} - \frac{1}{2}D_{nk} - AE_{nk} \right] \left[\left(\tfrac{1}{2}y\right)^{2(n+k)} \right] \tag{3-30}$$

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where

$$A = \frac{\gamma}{z} - \frac{1}{8}$$

(3-30-a)

and

$$a = \frac{W}{C_2 k}$$

(3-30-b)

Now, let us assume that

$$V_n(y) = \frac{1}{4} \sum_{k=0}^{\infty} a_{nk} \left(\frac{1}{z} y\right)^{2(n+k)}$$

(3-31)

The first derivative with respect to $y = kr$ can be expressed as

$$\frac{dV_n}{dy} = \frac{1}{4} \sum_{k=0}^{\infty} (n+k) a_{nk} \left(\frac{1}{z} y\right)^{2(n+k)-1}$$

(3-32)

and the second derivative with respect to $y = kr$ can be expressed as

$$\frac{d^2 V_n}{dy^2} = \frac{1}{4} \sum_{k=0}^{\infty} (n+k)(n+k-\frac{1}{z}) a_{nk} \left(\frac{1}{z} y\right)^{2(n+k)-z}$$

(3-33)

Substituting Eqs.(3-31) through(3-33) into the left-hand side of Eq.(3-30) yields

$$\sum_{k=0}^{\infty} \left\{ \left(\frac{1}{za}\right)^2 \left[\frac{1}{4}(n+k)(n+k-\frac{1}{z}) a_{nk} \left(\frac{1}{z} y\right)^{2(n+k-1)} \right. \right.$$

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$$+ \frac{1}{4} \frac{1}{y} (n+k) \, a_{nk} \left(\frac{1}{2} y \right)^{2(n+k)-1}$$

$$- \left(\frac{2n}{y} \right)^2 \frac{1}{4} \, a_{nk} \left(\frac{1}{2} y \right)^{2(n+k)} \right] + \frac{1}{4} a_{nk} \left(\frac{1}{2} y \right)^{2(n+k)} \qquad (3-34)$$

Eq.(3-34) must be equal the right-hand side of Eq.(3-30). Thus,

$$\left(\frac{1}{2a} \right)^2 (2n+k+1)(k+1) a_{n,k+1} + a_{nk} = - \left(\frac{1}{8} C_{nk} + \frac{1}{2} D_{nk} + A E_{nk} \right) \qquad (3-35)$$

where k = 0, 1, 2, 3 $\cdots\cdots\cdots$ (3-36)

If Eq.(3-35) is substituted into Eq.(3-36), then Eq.(3-35) can
be expressed as

$$a_{no} + \left(\frac{1}{2a} \right)^2 (2n+1)(1) \, a_{n1} = - \left(\frac{1}{8} C_{no} + \frac{1}{2} D_{no} + A E_{no} \right)$$

$$a_{n1} + \left(\frac{1}{2a} \right)^2 (2n+2)(2) \, a_{n2} = - \left(\frac{1}{8} C_{n1} + \frac{1}{2} D_{n1} + A E_{n1} \right)$$

$$a_{n2} + \left(\frac{1}{2a} \right)^2 (2n+3)(3) \, a_{n3} = - \left(\frac{1}{8} C_{n2} + \frac{1}{2} D_{n2} + A E_{n2} \right)$$

$$\vdots \qquad \qquad \vdots \qquad \qquad \vdots \qquad \qquad \vdots$$

$$\vdots \qquad \qquad \vdots \qquad \qquad \vdots \qquad \qquad \vdots$$

$$\vdots \qquad \qquad \vdots \qquad \qquad \vdots \qquad \qquad \vdots$$

$$a_{nk} + \left(\frac{1}{2a} \right)^2 (2n+k+1)(k+1) \, a_{n,k+1} = - \left(\frac{1}{8} C_{nk} + \frac{1}{2} D_{nk} + A E_{nk} \right)$$

$$(3-37)$$

The normal component of particle velocity is equal to zero on
the boundaries. From the application of the boundary condition
at the rigid boundary at r=a,

$$\frac{dV_n}{dy}\bigg|_{y=J'} = \frac{1}{4}\sum_{k=0}^{\infty}(n+k)a_{nk}\left(\tfrac{1}{2}j'\right)^{2(n+k)-1} = 0 \qquad (3-38)$$

where

$$J'(ka) = dJ/d(ka)$$

Substituting Eq.(3-36) into Eq.(3-38) yields

$$n a_{no}\left(\tfrac{1}{2}j'\right)^{2n-1} + (n+1)a_{n1}\left(\tfrac{1}{2}j'\right)^{2n+1}$$

$$+ (n+2)a_{n2}\left(\tfrac{1}{2}j'\right)^{2n+3} + \cdots + (n+k)a_{nk}\left(\tfrac{1}{2}j'\right)^{2(n+k-1)}$$

$$= 0 \qquad (3-39)$$

3. $\underline{U_n(y)}$

With Eqs.(3-9) through(3-29), Eq.(3-6) can be expressed as

$$\left[\left(\tfrac{1}{2a}\right)^2\left(\frac{\partial^2}{\partial y^2} + \frac{1}{y}\frac{\partial}{\partial y}\right) + 1\right]U_n(y) = \frac{1}{4}\sum_{k=0}^{\infty}\left[-\frac{1}{16}B_{n,k-4}\left(\tfrac{1}{2}y\right)^{2(n+k-2)}\right.$$

$$+ \frac{1}{4}G_{n,k-3}\left(\tfrac{1}{2}y\right)^{2(n+k-2)} - AE_{nk}\left(\tfrac{1}{2}y\right)^{2(n+k)}$$

$$\left. + \frac{1}{4}H_{n,k-1}\left(\tfrac{1}{2}y\right)^{2(n+k-2)} - \frac{1}{16}\Theta_{nk}\left(\tfrac{1}{2}y\right)^{2(n+k-2)}\right]$$

$$(3-40)$$

where, as before, $A = \gamma/2 - 1/8$ and $a = \dfrac{w}{c_s k}$

Now let us assume that

$$U_n(y) = \frac{1}{4} \sum_{k=0}^{\infty} b_{nk} \left(\tfrac{1}{2} y\right)^{2(n+k-2)} \tag{3-41}$$

The first derivative with respect to $y = kr$ can be expressed as

$$\frac{dU_n}{dy} = \frac{1}{4} \sum_{k=0}^{\infty} (n+k-2) \, b_{nk} \left(\tfrac{1}{2} y\right)^{2n+2k-5} \tag{3-42}$$

and the second derivative with respect to $y = kr$ can be expressed as

$$\frac{d^2 U_n}{dy^2} = \frac{1}{4} \sum_{k=0}^{\infty} (n+k-2)(n+k-\tfrac{5}{2}) \, b_{nk} \left(\tfrac{1}{2} y\right)^{2(n+k-3)} \tag{3-43}$$

Substituting Eqs.(3-41) through(3-43) into the left-hand side of Eq.(3-40) yields

$$\sum_{k=0}^{\infty} \left\{ \left(\tfrac{1}{2a}\right)^2 \left[(n+k)^2 - 4(n+k-1) \right] b_{nk} \left(\tfrac{1}{2} y\right)^{2(n+k-3)} \right. $$
$$\left. + b_{nk} \left(\tfrac{1}{2} y\right)^{2(n+k-2)} \right\} \tag{3-44}$$

Eq.(3-44) must equal the right-hand side of Eq.(3-40).
Thus,

$$\left(\tfrac{1}{2a}\right)^2 (n+k-2)^2 \, b_{n,k+1} + b_{nk} = \left(-\tfrac{1}{16} B_{n,k-4} + \tfrac{1}{4} G_{n,k-3}\right.$$

$$\left. + \tfrac{1}{4} H_{n,k-1} - \tfrac{1}{16} \Theta_{nk}\right) - A E_{n,k-2} \qquad (3\text{-}45)$$

where $k = 1, 2, 3 \quad \cdots \cdots \cdots$

If Eq.(3-45) is substituted into Eq.(3-36), then Eq.(3-45) can be expressed as

$$b_{n0} + \left(\tfrac{1}{2a}\right)^2 (n-2)^2 \, b_{n1} = -\tfrac{1}{16} \Theta_{n0}$$

$$b_{n1} + \left(\tfrac{1}{2a}\right)^2 (n-1)^2 \, b_{n2} = -\tfrac{1}{16} \Theta_{n1} + \tfrac{1}{4} H_{n0}$$

$$b_{n2} + \left(\tfrac{1}{2a}\right)^2 (n)^2 \cdot b_{n3} = -\tfrac{1}{16} \Theta_{n2} + \tfrac{1}{4} H_{n1} - A E_{n0}$$

$$b_{n3} + \left(\tfrac{1}{2a}\right)^2 (n+1)^2 \, b_{n4} = -\tfrac{1}{16} \Theta_{n3} + \tfrac{1}{4} H_{n2} - A E_{n1} + \tfrac{1}{4} G_{n0}$$

$$b_{n4} + \left(\tfrac{1}{2a}\right)^2 (n+2)^2 \, b_{n5} = -\tfrac{1}{16} \Theta_{n4} + \tfrac{1}{4} H_{n3} - A E_{n2} + \tfrac{1}{4} G_{n0} - \tfrac{1}{16} B_{n0}$$

$$\vdots$$

$$b_{nk} + \left(\tfrac{1}{2a}\right)^2 (n+k-2)^2 \, b_{n,k+1} = -\tfrac{1}{16} \Theta_{nk} + \tfrac{1}{4} H_{n,k-1} - A E_{n,k-2} + \tfrac{1}{4} G_{n,k-3} - \tfrac{1}{16} B_{n,k-4}$$

$$(3\text{-}46)$$

Application of the boundary condition at r=a yields

$$\frac{d \dot{U}_n}{d y}\bigg|_{y=J'} = \frac{1}{4}\sum_{k=0}^{\infty}(n+k-2)\dot{b}_{nk}\left(\tfrac{1}{2}j'\right)^{2n+2k-5} = 0 \qquad (3-47)$$

where

$$J'(ka) = dJ/d(ka)$$

Substituting Eq.(3-36) into Eq.(3-47) yields

$$(n-2)b_{no}\left(\tfrac{1}{2}j'\right)^{2n-5} + (n-1)b_{n1}\left(\tfrac{1}{2}j'\right)^{2n-3}$$

$$+ (n)b_{n2}\left(\tfrac{1}{2}j'\right)^{2n-1} + (n+1)b_{n3}\left(\tfrac{1}{2}j'\right)^{2n+1}$$

$$+ \cdots\cdots + (n+k-2)b_{nk}\left(\tfrac{1}{2}j'\right)^{2n+2k-5} = 0$$

$$(3-48)$$

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IV. CONCLUSIONS

Recall that we can compute a_{nk} from Eqs.(3-37) through(3-39) and Eqs.(3-14), (3-17), (3-20). Thus, with the use of Eq.(3-31), we can get V_n. From Eqs.(3-46) through(3-48) and Eqs.(3-11), (3-20), (3-23), (3-26), (3-29), we can compute b_{nk}. Thus, with the use of Eq.(3-14), we can get U_n. Therefore we can compute $V_n(y)$ and $U_n(y)$. Let us calculate the finite amplitude effects resulting from the nonlinear distortion of a forced radial mode. If we excite $a(0, m, 0)$ mode and obtain the pressure at the circumference, then $y = k_{om}a = j'_{om}$. Given this value of y, use of Eqs.(3-14), (3-17), (3-20) give the quantities C, D, E. From Eqs.(3-35) and (3-36) we can compute a_{nk}. Thus, we can get $V_n(y)$ from Eq.(3-31). Let us calculate the finite amplitude effects resulting from the nonlinear distortion of a forced sloshing mode. If we excite $a(1, m, 0)$ mode and obtain the pressure at the circumference, then $y = k_{1m}a = j'_{1m}$. Given this value of y, use of Eqs.(3-11), (3-20), (3-23), (3-26), (3-29) give the quantities B. E, G, H, O. From Eqs.(3-45) we can compute b_{nk}. Thus, we can get $U_n(y)$ from Eq.(3-41). Substituting n=0 at radial modes into $V_n(y)$ and $U_n(y)$ yields $V_0(y)=U_0(y)$. Therefore, with the use of Eq.(3-3), we can get the solution of Eq.(3-2).

A. RADIAL MODES(0, m, 0)

$$\frac{P_1}{\rho_0 C_0^2} = M J_0 (k_{om} r) \sin \omega t$$

then

$$\frac{P_2}{\rho_0 C_0^2} = 2M^2 V_0(y) \cos 2\omega t$$

so that

$$\left.\frac{|P_2|}{|P_1|}\right|_{r=a} = 2M V_0 (j'_{om}) / J_0 (j'_{om})$$

B. SLOSHING MODES(1, m, 0)

$$\frac{P_1}{\rho_0 C_0^2} = M J_1 (k_{1m} r) \cos\varphi \sin \omega t$$

then

$$\frac{P_2}{\rho_0 C_0^2} = M^2 \left[V_0(y) + V_1(y) \cos 2\varphi \right] \cos 2\omega t$$

so that

$$\left.\frac{|P_2|}{|P_1|}\right|_{r=a} = M \left[V_0(j'_{1m}) + V_1 (j'_{1m}) \cos 2\varphi \right] / \left[J_1(j'_{1m}) \cos\varphi \right]$$

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LIST OF REFERENCES

1. Blackstock, D. T., "Convergence of the Keck-Beyer Perturbation Solution for Plane Waves of Finite Amplitude in a Viscous Fluid", J. Acoust. Soc. Am., Vol. 39, No. 2, pp. 411-413, February 1966.

2. Coppens, A. B., and Sanders, J. V., "Finite-Amplitude Standing Waves Within Real Cavity", J. Acoust. Soc. Am., Vol. 58, No. 6, pp. 1133-1140, December 1975.

3. Maslen, S. H., and Moore, F. K., "On Strong Transverse Waves Without Shocks in a Circular Cylinder", Journal of the Aeronautical Sciences, Vol. 23, No. 6, pp. 583-593, June 1956.

4. Abramowitz, M. and Stegun, I. A., "Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables", National Bureau of Standards Applied Mathematics Series, No. 55, pp. 255, 360, December 1965.

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